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DECISION ANALYSIS OF STRATEGIC INTERACTION

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ABSTRACT

Strategic interactions occur when the outcomes of decisions depend on the actions of rivals, whose actions, in turn, depend on the rivals' beliefs about the decision-maker's decision. The result of a strategic interaction is an infinite outguessing regress in which the decision-maker attempts to guess the rivals' actions, which depend on the rivals' guesses of the decision-maker's decision, which depends on...

For decisions involving significant levels of complexity and uncertainty, the decision-maker's intuition alone may be inadequate, and quantitative modeling techniques may provide valuable assistance. Unfortunately, existing quantitative techniques for decision-making in the face of strategic interaction have significant shortcomings. This paper presents a quantitative decision-making technique that attempts to overcome many of these shortcomings. The technique prescribes a "best" decision alternative for the decision-maker, and, at the same time, allows the decision-maker to model descriptively the rivals' actions.

The centerpiece of the research is a model to overcome the outguessing regress of strategic interactions. This outguessing model is composed of three iterative steps: (1) maximize the decision-maker's expected utility based on utilities of outcomes given both the decision-maker's and the rival's actions and a probability

distribution on the rival's actions; (2) compute the decision-maker's probability distribution on the rivals' actions based on a model of the rival's action choice process that includes the rival's utilities of outcomes given joint decisions by rival and decision-maker and the rival's probability distribution on the decision-maker's decision; and (3) compute the rival's probability distribution on the decision-maker's decision based on the rival's perceptions of the decision-maker and the decision-maker's ranking of the decision alternatives. This is the step that breaks the infinite outguessing regress. The distribution is computed from a judgment about the rival's ability to "outguess" the decision-maker's preferences for alternatives.

1. INTRODUCTION

This paper presents, from a decision analysis perspective, a quantitative approach to decision-making with strategic interaction. Most decisions are made with some strategic interaction component, and in many important decisions, strategic interaction is the central issue. Strategic interaction occurs when a decision, or strategy choice, has as an important variable the actions of rivals, i.e., other individuals, organizations, or parties. The actions of these rivals, in turn, depend on the rivals' beliefs about the decision-maker's decision (or strategy) choice.

For decisions involving significant levels of complexity and uncertainty, the decision-maker's intuition alone may be inadequate, and quantitative modeling techniques may provide valuable assistance. Quantitative techniques can assist decision-makers by structuring the decision problem, identifying the important variables, incorporating and accounting for uncertainties and preferences, and allowing for the input of experts in different fields. The discipline of decision analysis has proven to be particularly well suited for these types of problems (Howard and Matheson, 1984). Unfortunately, existing quantitative techniques for decision-making in the face of strategic interaction have significant shortcomings. This research extends existing decision analysis techniques to strategic interactions in an attempt to overcome many of these shortcomings.

Research by Others

Research by others in the fields of decision analysis and game theory has led to the development of various quantitative techniques to support decision-makers when strategic interaction is important. The difficulty of applying the standard decision analysis approach to strategic interactions is in assessing a subjective probability distribution over the possible actions of the decision-maker's rivals. In assessing this probability distribution in a strategic interaction, the decision-maker quickly gets into the problem of an infinite "outguessing regress" where the likelihood of the rival's actions depends on the decision-maker's decision, which depends on the likelihood of the rival's actions, ad infinitum.

One of the more significant limitations of game theory, shared by both the complete and incomplete information approaches, is its jointly prescriptive nature, i.e., it assumes that both the decision-maker and the rival will indeed make the prescribed choices. Game theory provides no means to account for the behavior of rivals who do not follow the game-prescribed actions, which is usually the case in the real world. Although specific cases of what may be termed descriptive behavior have been treated in game theory, they are just that -- specific cases or types of descriptive behavior, which cannot be generalized.

Classical game-theory research has been heavily weighted towards games with "complete" information in which all players have full information about the other players' payoff functions, the physical facilities and strategies available to the other players, and the information other players possess. Unfortunately, most real-life situations do not meet these criteria. Development of Bayesian games with incomplete information is one attempt to cope with the unrealistic nature of the complete-information assumption of classical game theory (Harsanyi, 1982; Aumann, 1987). The most significant limitation of incomplete-information game theory is its assumption of common knowledge, which implies a common joint probability distribution on the state of information and values of both the decision-maker and the rival. Based on a few simple assumptions, the existence of this set of common knowledge can be proven, but the difficulties in assessing this common probability distribution appear insurmountable.

Scope and Organization

Section 2 sets forth the guiding principles of this research and outlines the technical approach, thereby setting the stage for the outguessing model -- the centerpiece of this research, which is presented in Sec. 3. Section 4 discusses in greater depth the assessment of the rivals' perceptions of the decision-maker. Section 5 is an example illustrating the application of the approach.

While the approach presented is designed to apply to all strategic interactions, this paper focuses on the most straightforward case of a one-time decision with a single rival choosing an action simultaneously with the decision-maker. Please see Strait (1987) for application of this approach to more complex strategic interactions, as well as additional examples and a realistic application to nuclear test ban treaty compliance evaluation.

Throughout this paper the term "rival" identifies the other individual or party whose interests are in conflict with the decision-maker's in a strategic interaction. For clarity, the rival's choices will be referred to as "actions," and the use of "decision" will be limited to the choice of the decision-maker.

2. PRESCRIPTIVE/DESCRIPTIVE APPROACH

This research attempts to capture the essence of strategic interactions. The approach is based on a set of intuitive principles that should appeal to the intuition of all decision-makers. The approach uses a decision analysis view to assist the decision-maker prescriptively, and then -- still in a decision analysis framework -- grapples with the problems of descriptively modeling the rival's actions.

Prescriptive Decision-Making

The prescriptive approach to decision-making followed by this research is that of the field of decision analysis. On a general level, this approach emphasizes capturing the structure of the problem and its relationships, as clearly presented by Howard (1968). On a more basic level, decision analysis treats uncertainty by means of subjective probability and treats the decision-maker's attitude toward risk through utility theory. The basics of the approach are usually expressed in five to seven simple axioms (for an example, see Savage (1972)).

The principles of decision analysis applied to a strategic interaction are distinctly asymmetric. The decision analysis expected utility maximization is clearly prescriptive from the decision-maker's viewpoint, i.e., the method prescribes a "best" decision alternative that the decision-maker should choose. In contrast, the rival's actions are always uncertain from the decision-maker's perspective and are reflected in a probability distribution. The derivation of this probability distribution is clearly descriptive in character, although it may assume the use of a prescriptive approach to action choice by the rival. Consequently, there is no concept, as in game theory, of solving for the rival's "best" action, checking it against the decision, and arriving at some sort of equilibrium.

When an outguessing regress occurs, the stumbling block to straightforward application of decision analysis techniques is the assessment of the decision-maker's probability distribution on the rival's actions. As pointed out earlier, the outguessing regress of strategic interactions precludes the direct assessment of this probability distribution. Consequently the decision-maker needs to take a broader view of computing the probability distribution on the rival's actions and use a model of the rival's action choice.

Descriptive Model of Rival's Actions

In developing an explicit model of the rival's action choice, the first consideration is the approach the rival will use to choose an action. This research is concerned with rivals who are in some sense decision-focused. These rivals are aware of the outguessing nature of problem and are attempting to account for it in choosing their action. The rival's approach could be either qualitative or quantitative. The exact nature of the decision-focused rival's action choice process is not significant. In fact, the rival could even be using the approach presented in this report. If the rival is using this approach, the decision-maker's assessment of the variables in her decision analysis may be affected, but her use of this approach is not.

The decision-maker and the rival both using this same approach is quite different from the jointly prescriptive nature of game theory. In both, the decision-maker's and rival's methods are the same. However, only in game theory is there an assumption that the actual choices (possibly randomized) of the rival and the decision-maker are part of the same equilibrium.

In modeling the action choice of a decision-focused rival in an outguessing regress, one simple guiding principle is dominant. This principle is:

> If the decision-maker prefers one alternative to a second alternative, then the decision-maker's probability distribution on the rival's actions should reflect a belief by the rival that the decision-maker is more likely to decide on the first alternative than to decide on the second alternative.

This principle ensures consistency among the decision, the decision-maker's beliefs, and the decision-maker's assessment of the rival's actions.

3. OUTGUESSING MODEL

The outguessing model reflects a three-step view of outguessing regress and follows the principles of the previous section. These steps are tightly bound in an iterative process, which results in an expected utility maximizing decision for the decision-maker. Although each step alone may not appear particularly powerful, together they provide a basis for a broader view of the decision analysis of strategic interaction. These three steps and their iterative nature are illustrated in Fig. 1.

(insert Fig. 1 here)

Model Logic

Following a decision analysis approach, the first step of the outguessing model is the decision-maker's expected utility maximization using a probability distribution on the rival's actions. The second of the three steps is the calculation of the decision-maker's probability distribution on the rival's actions based on a model of the rival's action choice. One model that meets the dominant principle for descriptively modeling the rival is a maximization of the rival's expected utility. This requires an assessment of the rival's utilities plus the rival's probability distribution on the decision-maker's decision.

The third step is the decision-maker's assessment of the rival's probability distributions on the decision-maker's decision. This distribution is dependent on the decision-maker's decision, which, of course, is not determined before completion of the analysis. This dependency is necessitated by the guiding principles in Sec. 2. This step breaks the infinite outguessing regress by analyzing the rival's perception of the decision-maker and the decision-maker's ranking of the decision alternatives. Specifically, the distribution is computed from a judgment about the rival's ability to outguess the decision-maker. Outguessing is defined as the rival's ability to rank the decision-maker's alternatives. This third step completes the iterative loop illustrated in Fig. 1.

Decision-maker's Expected Utility Maximization

Neglecting, for the moment, the difficulties in assessing the probability distribution on the rival's actions, the preferred decision is the decision alternative, out of the set of all feasible decision alternatives $\{d^1, d^2,..d^i,..d^n\}$, that maximizes the decision-maker's expected utility. The expected utility maximization can be expressed as follows:

$$\underset{d^i}{\overline{\text{Max}}^{1}} \int_a f(a|d^i)E[U|d^i,a] \quad (1)$$

where

a = rival's action,

$f(a|d^i)$ = decision-maker's probability density function on rival's action conditional on decision-maker's decision,

$E[U|d^i,a]$ = expected utility of decision d^i when rival chooses action a.

In Eq. (1) the decision-maker's probability distribution on the rival's action is dependent on the decision-maker's decision. This is due to the nature of strategic interaction and, particularly, the guiding principle for descriptively modeling the rival (see Sec. 2). The rival's action depends on the rival's view of the decision-maker's decision, and in an outguessing situation the decision-maker's decision and the rival's action are inextricably tied. This dependency between the rival's action and the decision assumes that if an alternative is the decision-maker's decision, then the rival will view that alternative beforehand as most likely to be chosen by the decision-maker.

Initially, this dependency may be somewhat confusing. It may appear to include the decision-maker's decision as a description of a state of the world upon which to condition the decision-maker's probability distribution on the rival's action. However, in actuality the decision-maker is only using the decision as a descriptor of her knowledge and utilities that make an alternative the recommended decision.

Rival's Expected Utility Maximization

The second step of the outguessing model computes a probability distribution on the rival's actions. This step is the maximization of the expected utility to the rival for outcomes of the strategic interaction. Rather than the standard decision analysis practice of directly assessing the probability distributions on the rival's actions, the approach is to assess the rival's utilities and then determine a probability distribution using these utilities.

By assessing the rival's utilities and calculating a probability distribution from them, the decision-maker can gain perspective on her interrelationship with the rival. While the decision-maker's and the rival's utilities for outcomes are related through the strategic interaction, the utility interrelationships are much clearer and more assessable than those of the alternative choice itself. From the decision-maker's perspective, the determination of the decision-maker's probability distribution on the rival's actions can be represented by Eq. (2):

$$f(a|d^i) = \int_{V_r}\int_{s_r} f(V_r)f(s_r)\ \delta[a - \max_{a^j}^{-1} \sum_{d_r} p_r(d_r|d^i)E[U_r|a^j,d_r,V_r,s_r]] \qquad (2)$$

where

$f(V_r)$ = decision-maker's probability density function on form of rival's utility function,

$f(s_r)$ = decision-maker's probability density function on rival's beliefs about the states-of-nature, e.g. probability of rain,

$p_r(d_r|d^i)$ = rival's probability distribution on decision-maker's decision conditional on decision-maker deciding d^i,

$E[U_r|a^j,d_r,V_r,s_r]$ = rival's expected utility for action a^j given decision d_r by the decision-maker, the rival's utility function, and the rival's beliefs about the states-of-nature ($U_r = V_r(a^j,d_r,s_r)$),

δ = unit impulse function.

The last half of the right side of the equation is the selection of the rival action that maximizes the rival's expected utility, from the decision-maker's perspective. The result of the rival's expected utility maximization is deterministic: i.e., a specific action, given a rival utility function and rival's beliefs about the states-of-nature. From the decision-maker's perspective, the rival's utility function, V_r, determines U_r, given the rival's action, the decision-maker's decision, and a state-of-nature.

The rival's probability distribution on the decision-maker's decision is also conditional on the decision-maker's actual decision. The dependency of the rival's probability reflects the nature of strategic interactions and the principle for descriptively modeling the rival. It is this interrelationship of the rival's beliefs and the decision-maker's decision that causes the decision-maker's probability distribution on the rival's actions to be dependent on the decision-maker's decision in Eq. (1) and Eq. (2).

The maximization of the rival's expected utility, while somewhat prescriptive in nature, is justified by the decision-maker assuming that the rival uses a decision-focused approach to action choice. Without such a prescriptive approach by the rival, an outguessing situation cannot occur. The approach is not jointly prescriptive in the game theory sense because the decision-maker doesn't settle on a single recommended rival action that the decision-maker feels the rival should and will choose, but rather only uses this methodology to capture all the elements of the rival's own action choice problem. There can be a significant degree of uncertainty by the decision-maker about important rival judgments, which also prohibits a jointly prescriptive interpretation.

Unfortunately, treatment of the decision-maker's decision in the model of the rival's action choice cannot be as straightforward as indicated above. Because of the outguessing regress, the rival's probability distribution on the decision-maker's decision in Eq. (2) presents assessment difficulties similar to those for the decision-maker's probability distribution on the rival's action in Eq. (1).

Rival's Perception of the Decision-maker

The third step of this approach is the modeling of the rival's perception of the decision-maker, to calculate the rival's probability distribution on the decision-maker's decision. The rival's probability distribution on the decision-maker's decision can be characterized by two components. The first of these is the rival's beliefs about the decision-maker's rank order for the decision alternatives. For example, the rival believes that d^x will be the decision-maker's first-ranked alternative and decision, d^z will be the decision-maker's second-ranked alternative, d^y will be the decision-maker's third-ranked alternative, and so on. The second component is the rival's probability distribution that an alternative of each particular rank will be the decision-maker's decision. For example, the rival believes there is probability p that the alternative the rival thinks is the decision-maker's wth-ranked alternative will actually be the decision-maker's decision; probability q that it will be the decision-maker's uth-ranked alternative, etc.

These two components comprise the rival's probability distribution on the decision-maker's decision as shown in Eq. (3):

$$p_r(d_r) = \sum_{h_r} p_r(h_r)\ \delta[d_r - R_r(h_r)] \tag{3}$$

where

$p_r(h_r)$ = rival's view of rank of alternative that is decision-maker's decision, $h_r \in \{1,2,...n\}$,

$R_r(h_r)$ = rival's estimate of decision-maker's decision alternative ranked h_r given the decision-maker's actual order R.

From the decision-maker's perspective, the rival's beliefs about the decision-maker's rank order do depend on the decision-maker's actual rank order, due to the outguessing nature of the problem. Therefore the decision-maker's probability distribution on the rival's beliefs will be conditional on the decision-maker's actual rank order R. However, from the principle for descriptively modeling the rival, this probability only measures the rival's uncertainty around the decision-maker's actual rank, and is therefore independent of the outguessing regress. Also, the rival's probability distribution that an alternative of a particular rank will be the decision-maker's decision, $p_r(h_r)$, can be independent of specific decision-maker's alternatives and therefore the rank order of specific alternatives and the rival's beliefs thereof. The degree to which this view of the rival's probability distribution on the decision-maker's decision has truly broken the outguessing regress will become particularly evident in the decision-maker's assessment of these two components, which is discussed more fully in Sec. 4.

Conditioning the rival's probability distribution on the decision-maker's decision in Eq. (2) on the two components of the rival's perception of the decision-maker yields the following equation for the decision-maker's probability distribution on the rival's actions:

$$p(a|R^i) = \sum_{R_r} \int_{p_r(h)} \int_{V_r} \int_{s_r} p(R_r|R^i) f(p_r(h_r))\ f(V_r) f(s_r)$$
$$\delta[a - \max_{a^j}{}^{-1} \sum_{d_r} p_r(d_r|R_r,h_r)\ E[U_r|a^j,d_r,V_r,s_r]] \qquad (4)$$

where

R^i = decision-maker's rank ordering of all decision alternatives such that the decision d^i is ranked first (as will be shown later, d^i uniquely determines R^i),

$p(R_r|R^i)$ = decision-maker's uncertainty about rival's view of decision-maker's rank ordering, conditional on actual rank ordering,

$f(p_r(h_r))$ = decision-maker's uncertainty about the rival's probability that any alternative of rank h_r will be the decision-maker's decision.

Assessment of the rival perceptions completes the three-step view of outguessing regress. However, the assessments of rival perceptions are conditioned on the decision-maker's actual ranking of alternatives. This ranking is the desired output of the outguessing model, not an input.

Ranking of Decision Alternatives

In a standard decision analysis without outguessing, determining the decision-maker's complete ranking is a simple extension of determining the decision-maker's preferred, expected utility maximizing alternative (e.g., what is the second, third,... alternative in terms of expected utility?). But in an outguessing regress, this ranking is more difficult. The determination of the decision-maker's preferred alternative is accomplished by a process that iteratively employs the three steps described previously.

This iterative ranking process begins with the ordering of the alternatives ranked last and next-to-last and continues by building up the decision-maker's rank order of alternatives. At each rank ordering decision, an expected utility maximization is performed on the reduced set by using the corresponding distribution on the rival's actions from the three steps of the outguessing model. This process continues until completion of the utility maximizing rank of all alternatives -- of which, barring ties, there will be only one.

It may be intuitive to view this ranking determination from a decision tree perspective. For example, assume there are four decision alternatives, d^1, d^2, d^3, and d^4. In the process of determining the preferred alternative a ranking of all four alternatives, R(4), will be determined. This ranking is done with a decision analysis expected utility maximization at each rank. Figure 2 illustrates these ranking decisions, ignoring all uncertainties. In Fig. 2, once the orders of all possible pairs -- d^2 vs. d^3, d^2 vs. d^4, d^3 vs. d^4, d^1 vs. d^3, d^1, vs. d^4, and d^1 vs. d^2 -- have been determined, the set of all possible R(2) has been pared to six possible rankings. The process then moves to the possible triplets, R(3), and then, finally, moves on to R(4). And once the ranking of all alternatives (R[4] in Fig. 2) is determined, the decision analysis is complete.

(insert Fig. 2 here)

4. ASSESSING THE RIVAL'S PERCEPTION

The ability of the outguessing model to characterize a strategic interaction and to completely break the infinite outguessing regress depends, to a large extent, on the assessment of the rival's perceptions, or ability to outguess, the decision-maker. Different assessment approaches are possible and several are presented in this section. The principle for descriptively modeling the rival, as described in Sec. 2, helps define the approach for assessing the rival's perception.

By that principle, the decision-maker's probability of a rival's perceived rank order of the decision-maker's alternatives, $p(R_r|R)$, should be greater for rival estimated rank orders, R_r, that more closely resemble the decision-maker's actual rank order, and should be greatest for $R_r = R$. The interpretations of "more closely resembles" may vary, and will be discussed more fully below. The principle also requires that the decision-maker's assessment of the rival's probability that an alternative of rank x is the decision-maker's decision, $p_r(h_r = x)$, be greater than or equal to the probability for an alternative of rank y, $p_r(h_r = y)$, for all x less than y.

In the context of the outguessing model, there are some additional, intuitively appealing desiderata for the rival's probability distribution on the rank of the alternative that is the decision-maker's decision. These desiderata imply that perfect perception by the rival of the decision-maker implies that the rival knows the decision-maker's first choice without doubt, regardless of the alternative chosen. Conversely, no perception of the decision-maker by the rival implies that the rival feels that all decision alternatives are equally likely to be chosen. Intermediate levels of perception or outguessing ability cannot be thought of in such simple terms; they can only be thought of by the rival as probability distributions $p_r(h_r)$ on rival's view of the the rank of alternative that is the decision-maker's decision. With perfect perception, $p_r(h_r)$ would be a single impulse-probability function; with no perception, $p_r(h_r)$ would be a uniform probability distribution.

The remainder of this section discusses several possible approaches that satisfy the principles for descriptively modeling the rival.

Single Outguessing Parameter

Although there may be other possible approaches, this research has developed one way of characterizing the rival's perception of the decision-maker using a single parameter. The single parameter characterizes the rival's perception of the decision-maker as the rival's outguessing ability g. This outguessing ability, g, is the probability that the rival would correctly guess the decision-maker's rank order of any two decision alternatives chosen at random. Note that this definition implies nothing about specific decision alternatives, due to the fact that the assessed probability deals only with two alternatives chosen at random. Also, of course, before completion of the decision analysis, not even the decision-maker knows her own rank order, which is the purpose and result of the analysis.

This probability of the rival correctly ranking the decision-maker's alternatives chosen at random, g, must be greater than or equal to 0.5 and less than or equal to 1.0. When $g = 0.5$, the rival has no outguessing ability and is equally likely to order correctly or incorrectly two alternatives chosen at random. A value of $g = 1.0$ signifies absolute outguessing ability by the rival and implies that the rival knows the decision-maker's preferred alternative out of any two chosen at random. A value of g (i.e., a probability) of less than 0.5 would violate the guiding principles for descriptively modeling the rival. A value of less than 0.5 implies either that the rival is not attempting to outguess the decision-maker or that the decision-maker has specific knowledge of the rival's beliefs or actions, which transcends outguessing.

The rival's beliefs about the rival's own ability to outguess the decision-maker can be characterized separately by g_r, where g_r is defined similarly to g. The decision-maker can then assess the probability distribution $f(g_r)$. The probability distribution on the rival's beliefs about the decision-maker's rank ordering $p(R_r|R)$ is determined by g -- how well the decision-maker thinks the rival can outguess her. The rival's probability distribution on a decision-maker's alternative, given a rival perceived ranking $p_r(h_r)$, is determined using g_r, a reflection of the rival's uncertainty about the rival's own perceptions.

The determination of $p(R_r|R)$ from g and $p_r(h_r)$ from g_r both require an assumption about the rival's process for estimating the decision-maker's ranking. One possible assumption is that the rival determines R_r by considering each decision alternative in a random order and then comparing it with the best the rival has already considered. This process is continued until all alternatives are considered and the first-ranked one is determined. Those not first are then considered, by the same process, for the second rank. The process is continued until the entire rank order is determined.

The single-outguessing-parameter method of assessing the rival's perception of the decision-maker has many advantages. Defining the outguessing parameter, g, as a probability for two randomly selected alternatives, avoids any hint of an outguessing regress in the assessment. The determination from g_r of the rival's probability, $p_r(h_r)$, that an alternative is the decision-maker's h_rth-ranked alternative will be the decision-maker's decision is designed so that if the decision-maker and the rival have identical beliefs about the rival's outguessing ability (i.e., $g = g_r$), then the decision-maker's probability of the rival estimating the decision-maker's ranking of an alternative as being first is exactly equal to the decision-maker's expected probability the rival assigns to that alternative being the decision-maker's decision. The number of assessments is certainly reasonable, only g and $f(g_r)$. These assessments are also very tractable because g is a single probability and $f(g_r)$ is a probability distribution of a single variable.

The single-parameter approach by its very nature is not able to account for variations in the rival's perceptions for different decision-maker's alternatives. This ability may be important when particular alternatives appear, from the rival's perspective, to be significantly more likely than others to be the decision-maker's decision. It is also important when alternatives represent discrete points on a continuum. The detailed assessment approaches in the next subsection overcome this difficulty, but in doing so, they sacrifice some of the other benefits of the single-parameter approach.

Detailed Assessments

Detailed approaches to assessing the rival's perceptions of the decision-maker may overcome the weaknesses of the single-outguessing-parameter approach. Detailed approaches are, however, significantly less practical because of the number of assessments, the nature of the variables to be assessed, and the complexity of the calculations they entail.

There are various levels of detailed assessments. The most basic level would be to assess $p(R_r|R)$ and $f(p_r(h_r))$ directly for each iteration of the outguessing model. Except for the very simplest of problems, such an approach would require an extraordinary number of assessments, and $f(p_r(h_r))$ is a probability distribution on a probability distribution and difficult to assess. Assessments at this level would be capable of capturing all of the variations in the rival's perceptions according to the specific decision-maker's alternatives. There is no built-in consistency check between the decision-maker's probability distribution on the rival's actions and the decision-maker's assessment of the rival's beliefs.

There are at least two possibilities for less detailed approaches that still allow for variations in the rival's perceptions without sacrificing as much as the most basic level. These two approaches require assessments of only pair-wise comparisons and assume a model of the rival's process for estimating the decision-maker's ranking. This assumed model of the rival's process could be the same as that otherwise used in the single-parameter approach.

The more detailed of these approaches is to assess the probability of the rival's correctly estimating the decision-maker's ranking of two alternatives, given those alternatives and given the decision-maker's rank of those alternatives. Thus the decision-maker would have to make two assessments for each possible pair of alternatives. These probability assessments must both be greater than 0.5 to satisfy the guiding principles described in Sec. 2. The calculations for determining $p(R_r|R)$ and $p_r(h_r)$ would be significantly more involved than calculations for the single parameter, but they follow the same conceptual approach and may be practical. The determination of $f(p_r(h_r))$ would require assessments of the rival's beliefs concerning the rival's outguessing ability for each pair and ordering. This is a significant, but manageable, task. The approach does not have a built-in consistency between the decision-maker's probability distribution on the rival's actions and the decision-maker's assessment of the rival's beliefs about the decision-maker's decision.

Another approach would be to assess the probability of the rival correctly ranking two alternatives, given those alternatives, but not given the decision-maker's ranking. This approach has similar advantages and disadvantages to the one described in the preceding paragraph, but it sacrifices some flexibility in modeling the rival's perceptions for improvements in other areas. This approach may be particularly useful for decision problems with a discretized continuum of decision alternatives. In such a problem, it is probably reasonable to assume that the rival is less likely to rank correctly two alternatives adjacent on the continuum than two alternatives separated by a third alternative on the continuum. This may be true regardless of the eventual decision-maker's ranking.

5. EXAMPLE

This example presents the decision analysis of a simple strategic interaction using the outguessing model with the single outguessing parameter approach. Any uncertainties about the form of the rival's utility function, V_r, and beliefs about the states-of-nature, s_r, are ignored. This example illustrates only a small fraction of the power of the approach. One of the strengths of the approach is its ability to handle a large number of variables and wide ranges of uncertainty. The example presented here is a very basic problem, lacking most of the uncertainty inherent in even the most simple real-life strategic interactions.

Problem Description

Valentine's Day is quickly approaching and the decision-maker must decide what gift to buy her beau for the special day. The alternatives, as the decision-maker sees them, are to buy: (1) just a card; (2) a small token; or (3) that extra special something of a more personal and expensive nature. The decision-maker's relative utility for each of these alternatives depends on the gift her beau gives her. The decision-maker does not wish to be perceived as more or less serious about the relationship than her beau (e.g., she doesn't want to present him with silk pajamas in exchange for a goofy card from him). The decision-maker believes that her beau (i.e., the rival in this formulation) has the same gift alternatives. The decision-maker cannot, of course, discuss the gift issue with the beau since this would violate the spirit of gift-giving. The decision-maker has assessed the utilities in Fig. 3 for the possible outcomes.

(insert Fig. 3 here)

As these utilities indicate, the decision-maker would like to see the relationship become more serious and would prefer to exchange extra special gifts. But at the same time she does not want to pressure her beau away by being more serious than he is ready to be. The beau's utilities for possible outcomes are not the same as the decision-maker's. He is apprehensive about a commitment, and therefore prefers to exchange less serious gifts.

Using the single-outguessing parameter, the decision-maker does need to assess the beau's ability to outguess her, g. The decision-maker believes that $g = 0.8$, i.e., the rival would correctly guess her ranking of any two alternatives chosen at random with probability 0.8. The decision-maker also needs to assess how well the beau believes he can outguess her. Assume that the decision-maker believes the rival (beau) also will think his outguessing ability is $g_r = 0.8$.

Outguessing Model

Following the steps shown in Fig. 1 counterclockwise, the first iteration of the model is to rank all possible last and next-to-last pairs. This requires three separate decisions, because there are three possible next-to-last/last pairs to be ranked. For example, the decision tree in Fig. 4 shows the choice between alternative rankings of the card(c) and extra(e) alternatives. In that figure the decision-maker prefers extra(e) over card(c) (i.e., e/c) ranking with an expected utility 8.4. The probability distribution on the rival's perception of the decision-maker's ranking, $p(R_r|R^i)$ is calculated from the rival's outguessing ability, $g = 0.8$. For the last/next-to-last decisions, $p(R_r|R^i)$ is rather simple in form: either the rival correctly outguesses the decision-maker, probability 0.8 equal to g, and the rank as perceived by the rival is the same as the decision-maker's rank; or the rival fails at outguessing and the rank as perceived by the rival is exactly the opposite of the actual rank.

(insert Fig. 4 here)

For each perceived ranking by the rival, R_r, a rival action is determined by maximization of the rival's expected utility. This expected utility maximization requires a probability distribution by the rival on the decision alternatives. There are six rival decisions of concern, one for each possible ranking of the decision-maker's next-to-last/last choices. For example, in the decision tree in

Fig. 5, the rival believes that the decision-maker prefers e to c with probability 0.8. These probabilities depend on both R_r and g_r. The rival's expected value maximizing action is c_r with expected value 7.2 = (8)(0.8) + (4)(0.2). Accordingly, c_r is the first and fourth entries in the rival action column of Fig. 4. This then allows calculation of the expected utilities in that figure, and the determination the preferred ranking of possible last and next-to-last pairs.

(insert Fig. 5 here)

Now the final ranking can be completed because the choice of any decision alternative now defines a complete ranking: i.e., if token(t) is the preferred decision alternative, then the complete ranking is t/e/c, since e/c is the next-to-last/last ranking from Fig. 4. Figure 6 illustrates this top-level decision tree. Once again, the probability distribution on the rival's perceived ranking of the decision-maker's alternatives, $p(R_r|R^i)$, is determined by actual decision-maker ranking and the rival's outguessing ability g.

(insert Fig. 6 here)

The next step is to determine a rival expected utility maximizing action for each R_r. Since this is the top-level decision, there is only one decision-maker's decision tree, but there are many more possible rankings, R_r. For example, in Fig. 7, given R_r = c/t/e, the rival prefers c_r with an expected utility of 7.3 = (0.69)(8) + (0.25)(6) + (0.06)(4). This is reflected by the fifth entry in the rival's action column of Fig. 6.

(insert Fig. 7 here)

In a manner similar to the relation of Fig. 4 and Fig. 5, Fig. 7 determines the rival action for each rival perceived ranking, R_r, on Fig. 6, allowing the completion of the expected utilities column of that figure and the determination of the utility maximizing decision as token(t) with an expected utility of 7.6.

Sensitivity Analysis

To develop some additional insights into the outguessing model and its application, this subsection will investigate the sensitivity of the gift-giving example to a number of the decision-maker assessments. By using the single parameter approach this task is rather simple. Perhaps the most relevant assessments to examine are those of the rival's outguessing ability.

As the rival's outguessing ability g increases from 0.8 to 0.9, the decision-maker's preferred alternative changes from token to extra. In this example, there is a general tendency of the expected utility for each gift choice to increase with the rival's outguessing ability. This is expected due to the commonality of the decision-maker's and rival's utilities and their preference for exchanging similar gifts. In fact, if the rival has perfect outguessing ability ($g = 1.0$), the decision-maker's expected utility is maximized for each alternative. This is because of the character of the particular example having multiple, pure-strategy Nash equilibria and the guiding principles discussed in Sec. 2 for descriptively modeling the rival. If the decision-maker gives an extra gift, the rival also prefers to give an extra gift. However, if the rival were using this decision analysis approach, the rival would be driven, as outguessing ability increases, to the Nash equilibrium of exchanging tokens. Unfortunately, the guiding principles noted above are very compelling, and there appears to be no solution to this quandary other than to caution the decision-maker and the decision analyst in the assessment of the rival's outguessing ability.

An important factor in any strategic interaction is the decision-maker's uncertainty about the rival's values and state of information. In this example, as the decision-maker's uncertainty about the rival's utilities increases, the decision-maker's expected value decreases. This trend would be expected because information (i.e., lack of uncertainty) generally has value. The decision also switches from token to extra as uncertainty grows reflecting the structure of the decision-maker's and the rival's utilities. This demonstrates the importance of modeling all the decision-maker's uncertainty in a strategic interaction, an advantage of decision analysis and the approach developed in this research.

6. SUMMARY

This paper presented the results of research to develop a new quantitative technique for decision-makers faced with a strategic interaction. The approach is decision analytic in nature, and it assists the decision-maker in choosing a "best" decision to maximize her expected utility while descriptively modeling the rival's action. The approach intuitively captures the essence of strategic interactions, at the same time allowing the decision-maker to reflect in the analysis all of her knowledge of the decision problem.

An outguessing model is used to overcome the outguessing regress of strategic interactions. This outguessing model is composed of three iterative steps: (1) maximize the decision-maker's expected utility based on utilities of outcomes given both the decision-maker's and the rival's actions and a probability distribution on the rival's actions; (2) compute the decision-maker's probability distribution on the rivals' actions based on a model of the rival's action choice process that includes the rival's utilities of outcomes given joint decisions by rival and decision-maker and the rival's probability distribution on the decision-maker's decision; and (3) compute the rival's probability distribution on the decision-maker's decision based on the rival's perceptions of the decision-maker and the decision-maker's ranking of the decision alternatives. This is the step that breaks the infinite outguessing regress. The distribution is computed from a judgment about the rival's ability to "outguess" the decision-maker's preference for alternatives.

This decision analysis approach can be applied to simple as well as more complex strategic interactions. Its application is relatively practical in terms of structuring the decision, assessing the important variables, and calculating the recommended decision. Hopefully, this research will serve to improve and expand the application of decision analysis to important real-life decision problems involving strategic interactions.

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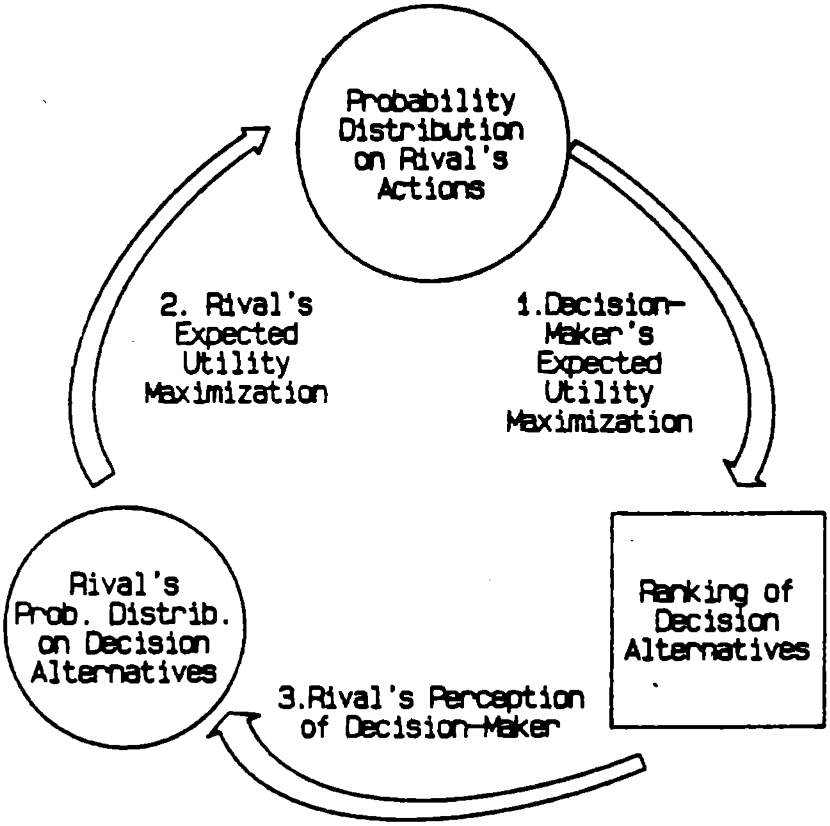
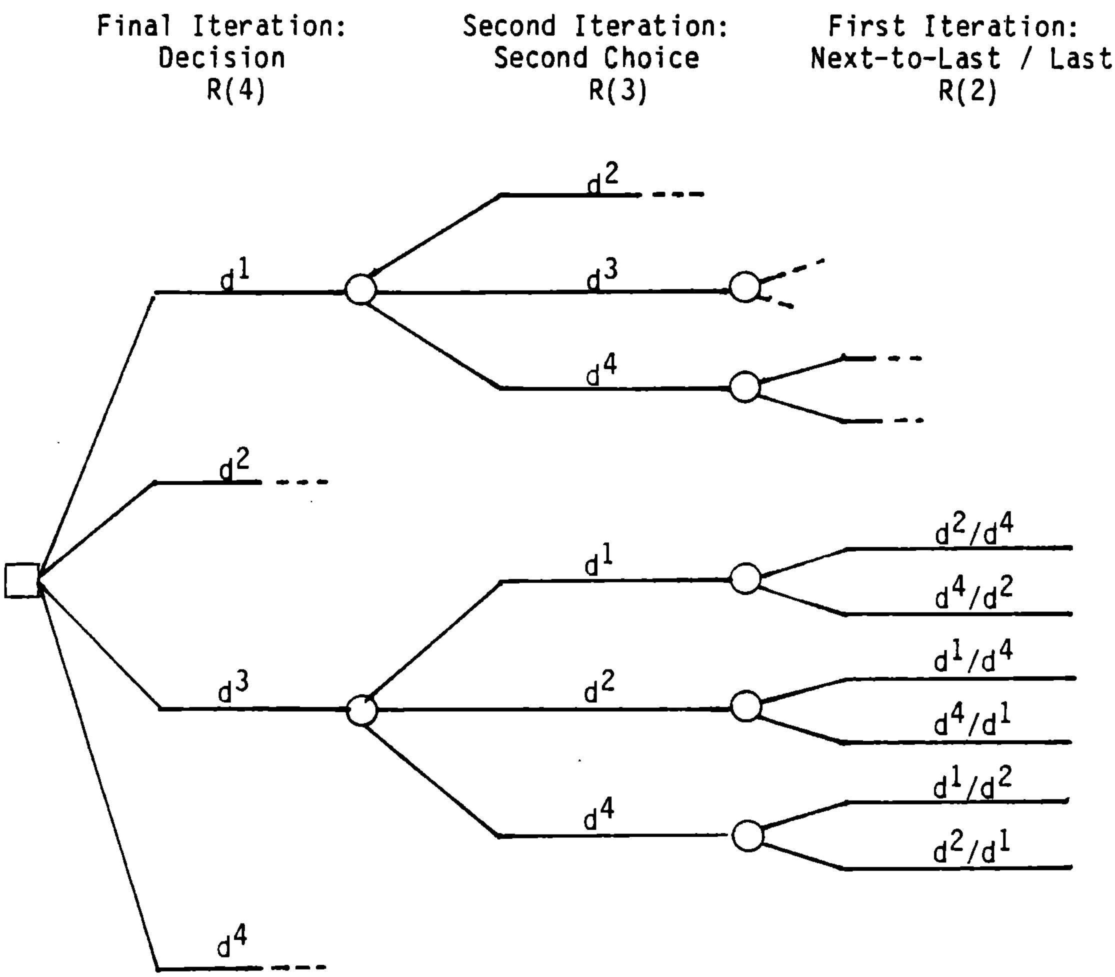

Probability Distribution on Rival's Actions
1.Decision-Maker's Expected Utility Maximization
Ranking of Decision Alternatives
3.Rival's Perception of Decision-Maker
Rival's Prob. Distrib. on Decision Alternatives
2. Rival's Expected Utility Maximization

Figure 1



Final Iteration:
Decision
R(4)
Second Iteration:
Second Choice
R(3)
First Iteration:
Next-to-Last / Last
R(2)
d^1
d^2
d^3
d^4
d^2
d^3
d^4
d^1
d^2
d^4
d^2/d^4
d^4/d^2
d^1/d^4
d^4/d^1
d^1/d^2
d^2/d^1

Figure 2

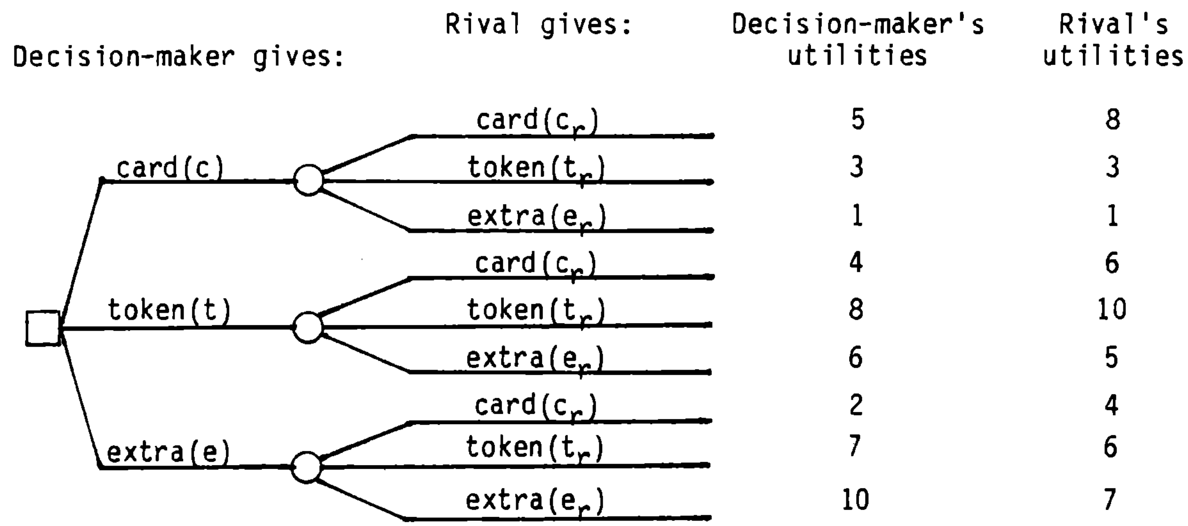


Figure 3

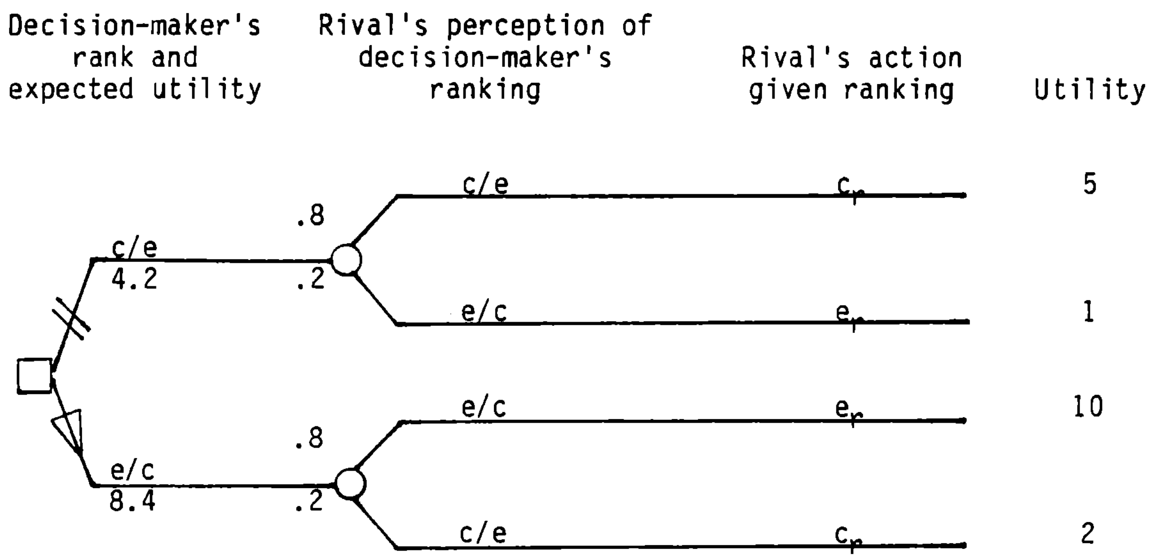


Figure 4

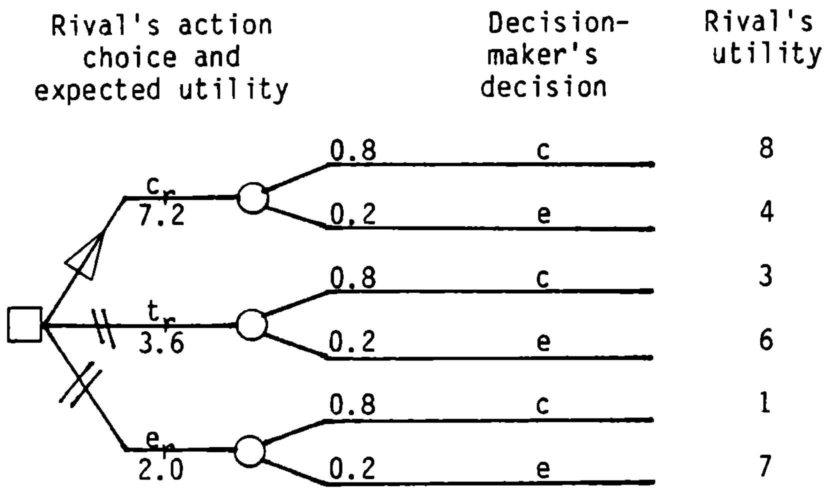


Figure 5

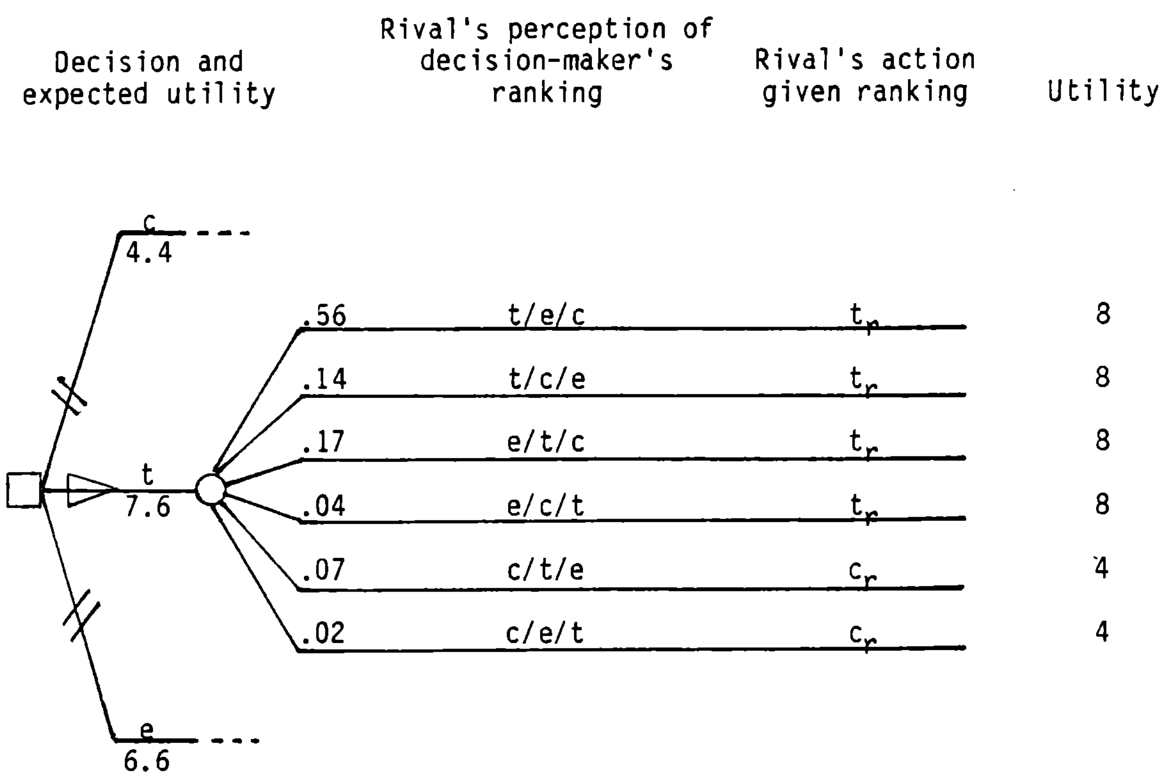


Figure 6

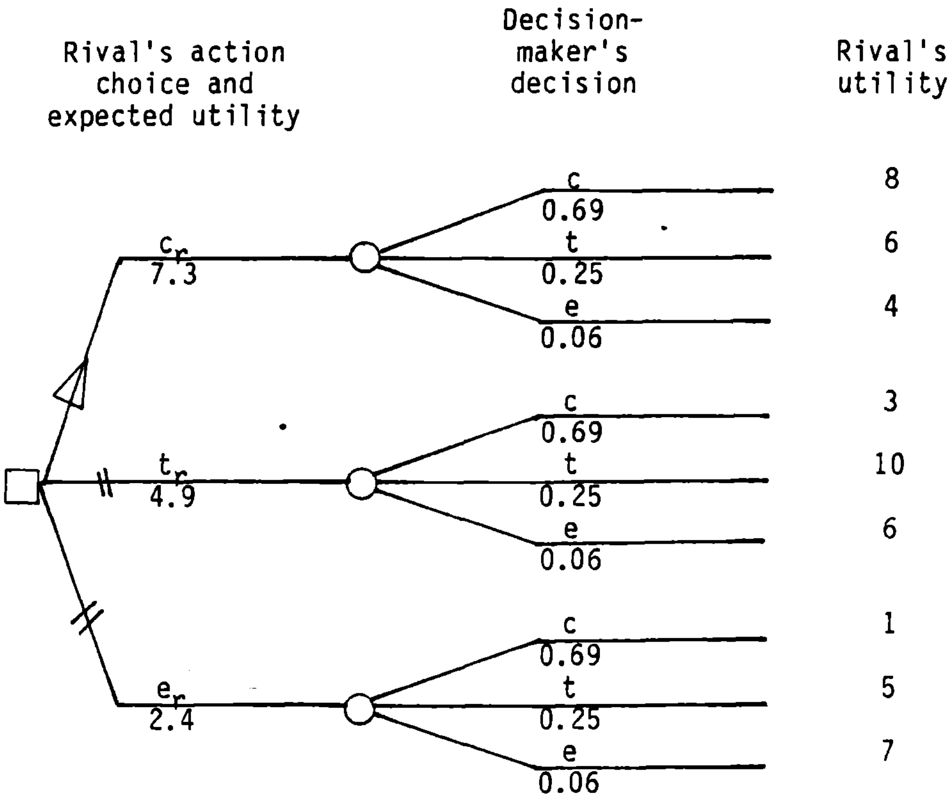

Rival's action choice and expected utility
Decision-maker's decision
Rival's utility
c_r
7.3
t_r
4.9
e_r
2.4
c
0.69
t
0.25
e
0.06
8
6
4
3
10
6
1
5
7

Figure 7